

**Benha University**  
**Faculty Of Engineering at Shoubra**



**ECE 122**  
**Electrical Circuits (2)(2017/2018)**  
**Lecture (01)**  
**Series Resonance**

**Prepared By :**

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# Resonance

Reference : Chapter (21)  
Circuit Analysis – Theories and Practice (Robinson & Miller)

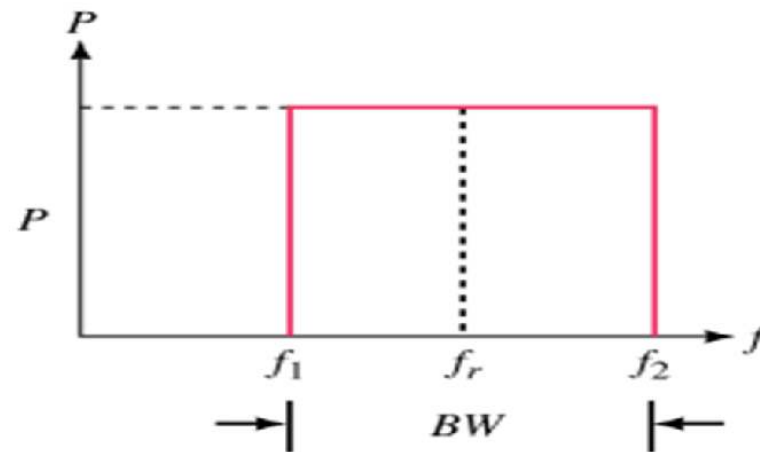
Circuits with both inductance and capacitance can exhibit a property called “**Resonance**” which is important in many applications

- Resonance is the basis for frequency selectivity in communication systems
- The ability of a radio or TV receiver to select a certain frequency (station) and at the same time eliminate frequencies from other stations is based on the principle of resonance

**In this section**  
We will observe how resonant circuits are able to pass a desired range of frequencies from a signal source to a load.

# Resonance

- ✓ In order to obtain all the transmitted energy for a given radio station or television channel, we would like a circuit to have the frequency response shown in Figure.



(a) Ideal frequency response curve

$f_r$  : center frequency = station carrier frequency

**BW: bandwidth** of the station = The difference between the upper and lower frequencies that we would like to pass

**A circuit having an ideal frequency response would pass all frequency components in a band between  $f_1$  and  $f_2$  , while rejecting all other frequencies.**

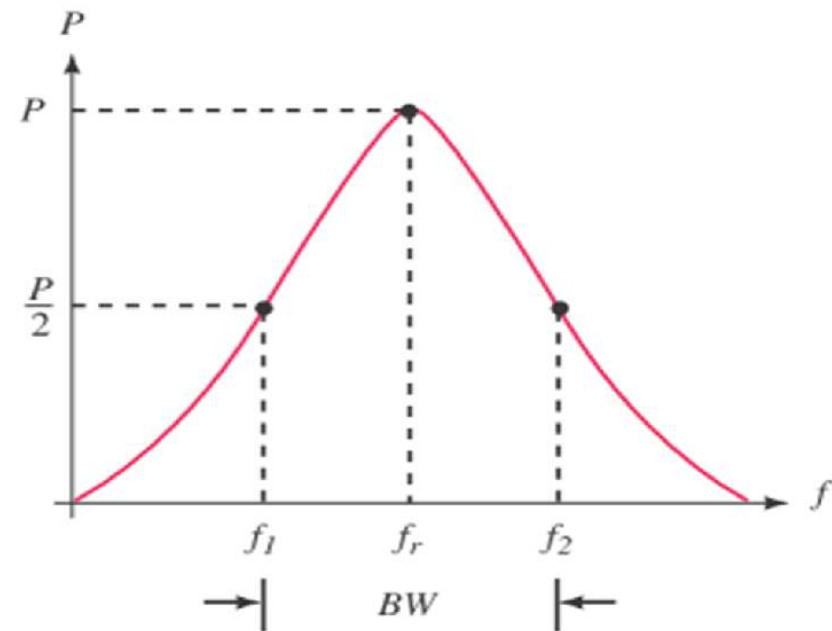
# Resonance

Whereas there are various configurations of resonant circuits, they all have several common characteristics.

1. The resonant circuit consists of at least an **inductor** and a **capacitor** together with a **voltage or current source**.
2. Have a bell-shaped response curve centered at a resonant frequency as in shown in figure

3. This curve indicates that power will be a maximum at  $f_r$  and varying the frequency in either direction results in a reduction of the power.

The bandwidth = the difference between the half-power points on the response curve of the filter.



(b) Actual response curve of a resonant circuit

## Series Resonance

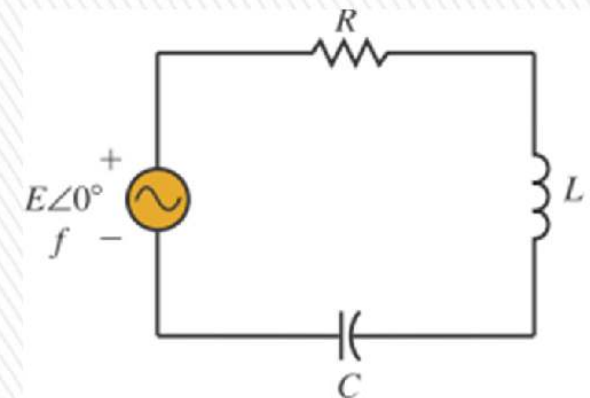
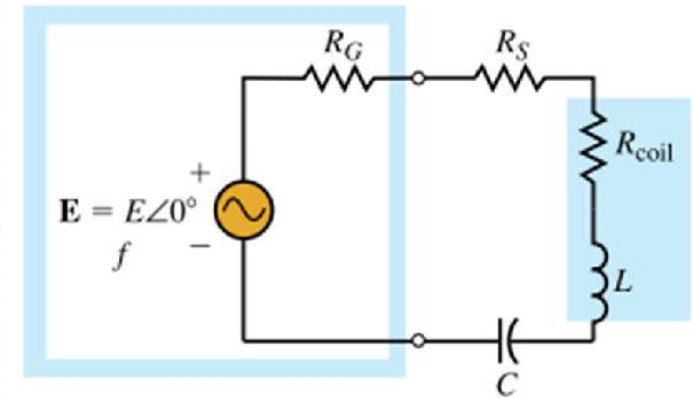
- $R_G$  : Generator resistance
- $R_S$  : Series resistance
- $R_{coil}$ : Inductor coil resistance

In this circuit, the total resistance is expressed as

$$R = R_G + R_S + R_{coil}$$

The total impedance is given by:

$$\begin{aligned} Z_T &= R + jX_L - jX_C \\ &= R + j(X_L - X_C) \\ &= R + j\left(\omega L - \frac{1}{\omega C}\right) \end{aligned}$$



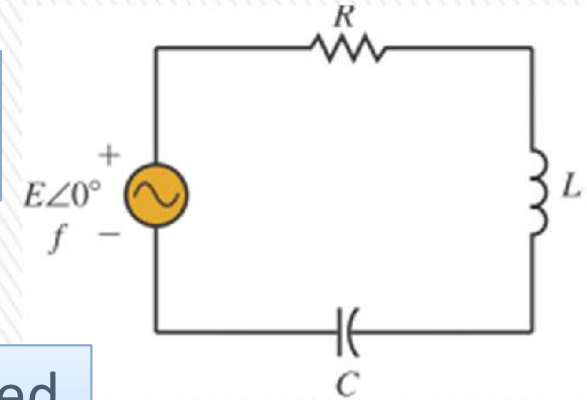
**Resonance** is a condition in an *RLC* circuit in which the capacitive and inductive reactances are equal in magnitude, thereby resulting in a purely resistive impedance.

## Series Resonance

By setting the reactance of the capacitor and inductor equal to one another, the total impedances given by:

$$Z_T = R$$

The value of  $\omega$  that satisfies this condition is called the resonant frequency ( $X_L = X_C$ )



$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega_s = \frac{1}{\sqrt{LC}} \quad (\text{rad/s})$$

$$f_s = \frac{1}{2\pi\sqrt{LC}} \quad (\text{Hz})$$

OR

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

## ANALYSIS OF SERIES RLC CIRCUITS

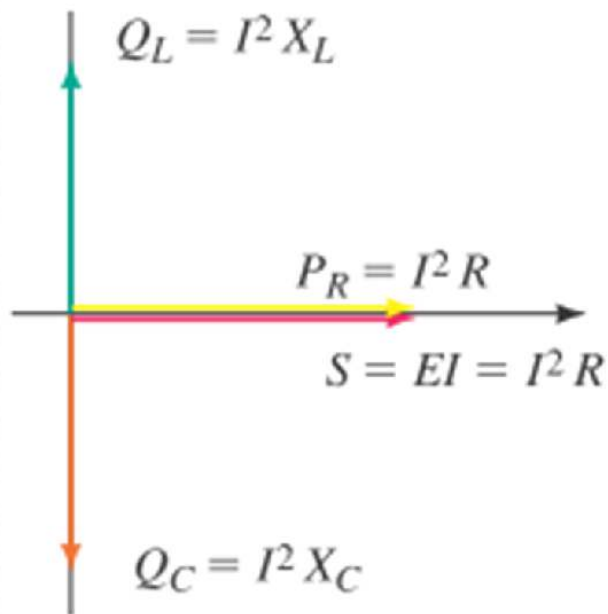
At resonance the total impedances given by:

$$Z_T = R$$

At resonance, the total current in the circuit is determined from Ohm's law as

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{E\angle 0^\circ}{R\angle 0^\circ} = \frac{E}{R}\angle 0^\circ$$

The voltage across each of the elements in the circuit as follows:



$$\mathbf{V}_R = IR\angle 0^\circ$$

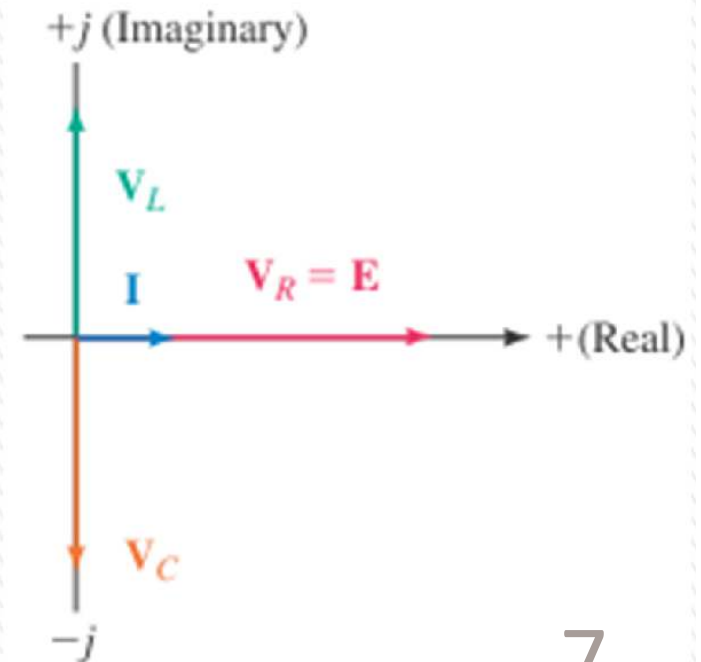
$$\mathbf{V}_L = IX_L\angle 90^\circ$$

$$\mathbf{V}_C = IX_C\angle -90^\circ$$

$$P_R = I^2 R \quad (\text{W})$$

$$Q_L = I^2 X_L \quad (\text{VAR})$$

$$Q_C = I^2 X_C \quad (\text{VAR})$$



## Impedance of a Series Resonant Circuit **versus** Frequency

Because the impedances of (L and C) are dependent upon frequency, the total impedance of a series resonant circuit must similarly vary with frequency

$$\begin{aligned}Z_T &= R + j\omega L - j\frac{1}{\omega C} \\ &= R + j\left(\frac{\omega^2 LC - 1}{\omega C}\right)\end{aligned}$$

Impedances Magnitude:

$$Z_T = \sqrt{R^2 + \left(\frac{\omega^2 LC - 1}{\omega C}\right)^2}$$

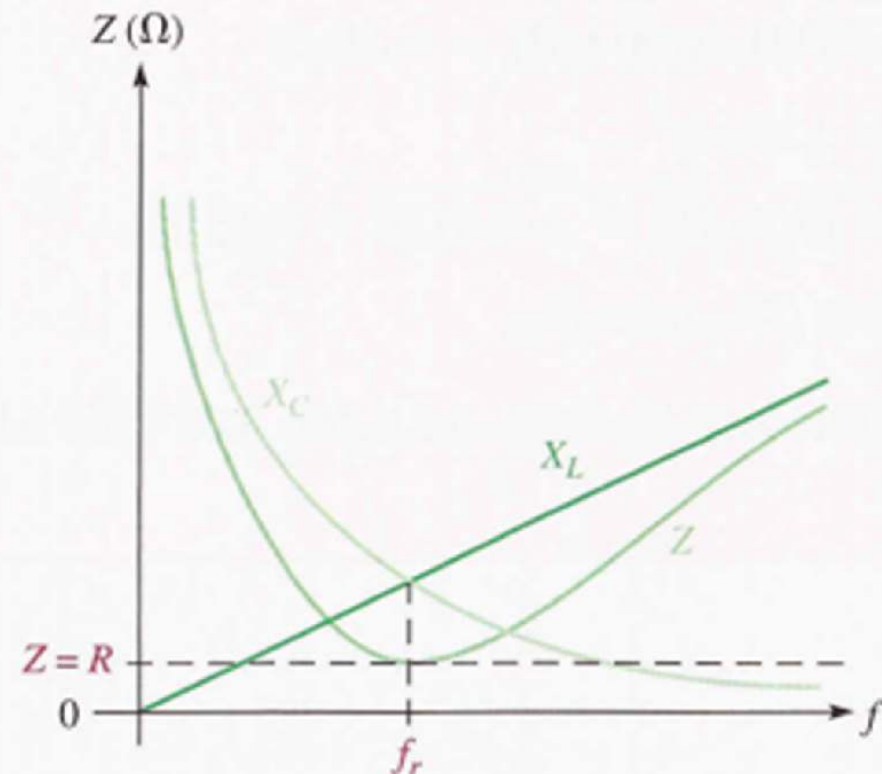
Impedances Angle:

$$\theta = \tan^{-1}\left(\frac{\omega^2 LC - 1}{\omega RC}\right)$$

When  $\omega = \omega_s$ :

$$Z_T = R$$

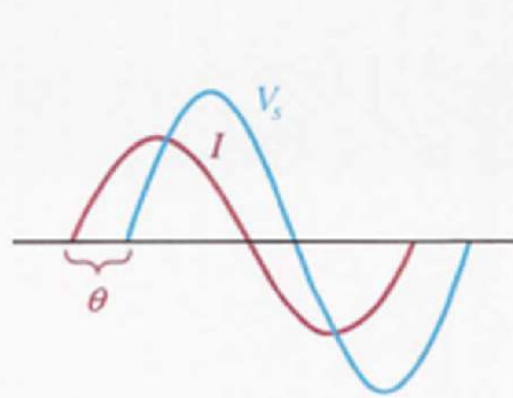
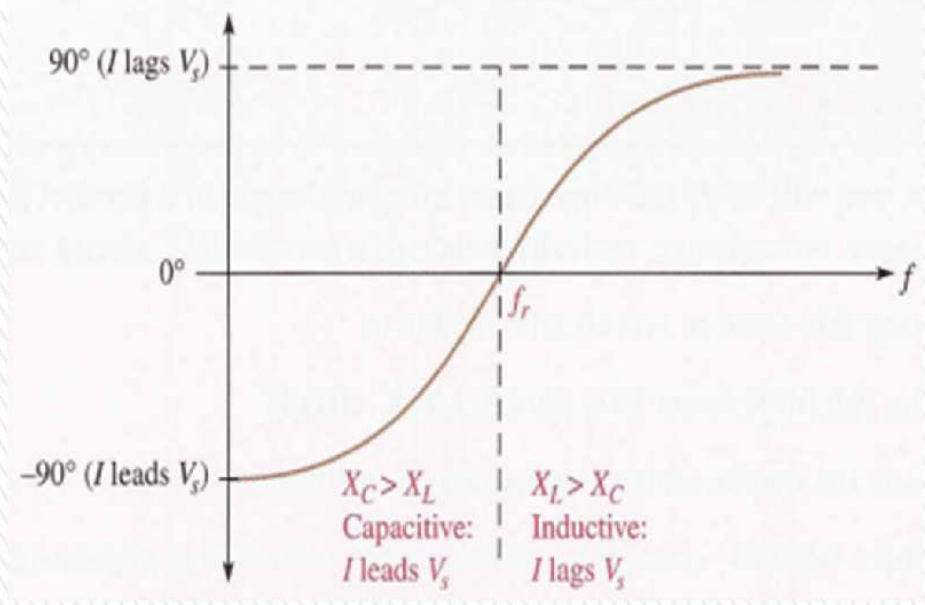
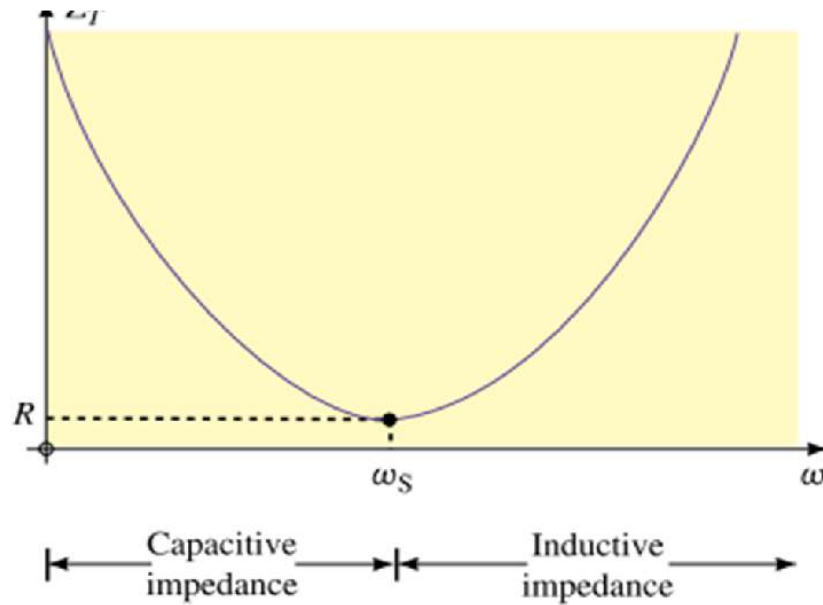
$$\theta = \tan^{-1}0 = 0^\circ$$



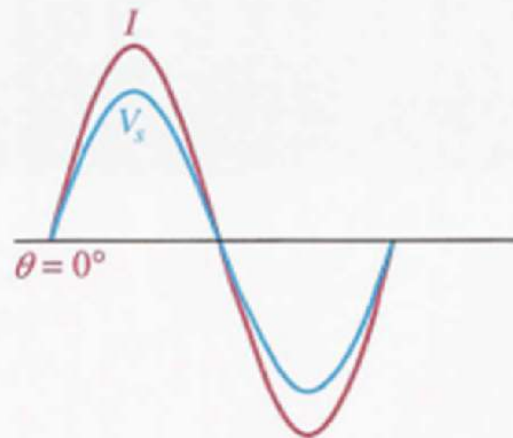


# Impedance of a Series Resonant Circuit **versus** Frequency

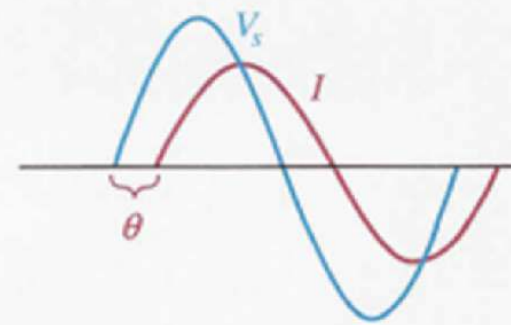
**FIGURE 21-7** Impedance (magnitude and phase angle) versus angular frequency for a series resonant circuit.



(a) Below  $f_r$ ,  $I$  leads  $V_s$ .



(b) At  $f_r$ ,  $I$  is in phase with  $V_s$ .



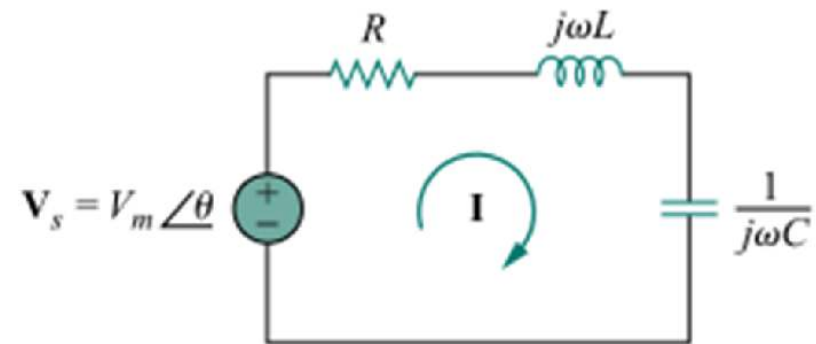
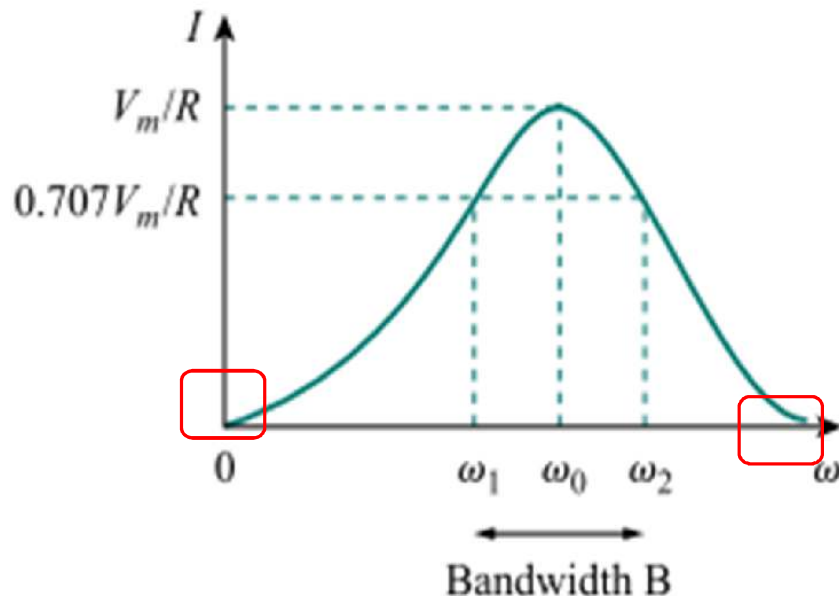
(c) Above  $f_r$ ,  $I$  lags  $V_s$ .

## Current and Power in a Series Resonant Circuit

In this section, we examine how current and power are affected by changing the frequency of the voltage source.

Applying Ohm's law gives the magnitude of the current at resonance as

$$I_{\max} = \frac{V_m}{R}$$



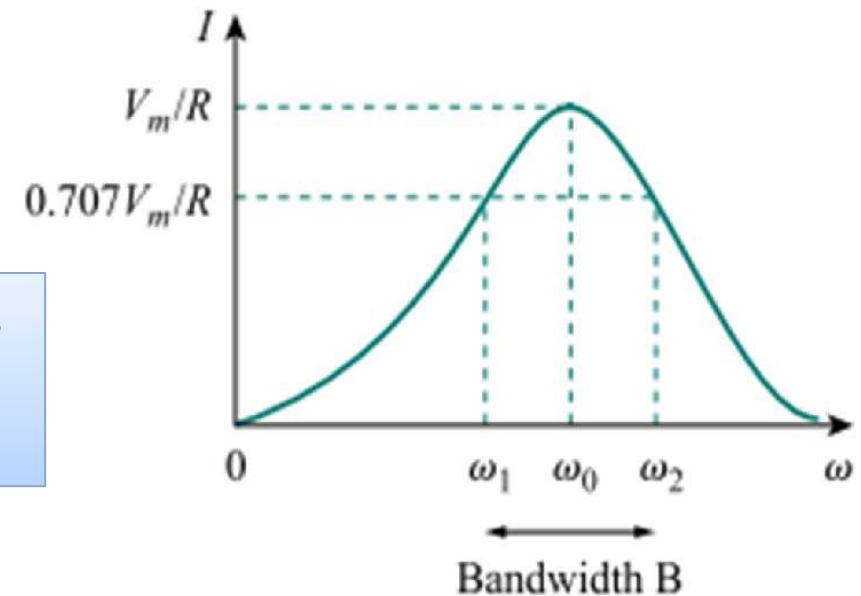
The frequency response of the circuit's current magnitude

$$I = |\mathbf{I}| = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

For all other frequencies, the magnitude of the current will be less than  $I_{\max}$  because the impedance is greater than at resonance.

## Current and Power in a Series Resonant Circuit

Since the current is maximum at resonance, it follows that the power must similarly be **maximum** at **resonance**.



- At certain frequencies  $\omega = \omega_1, \omega_2$ , the dissipated power is half of that max

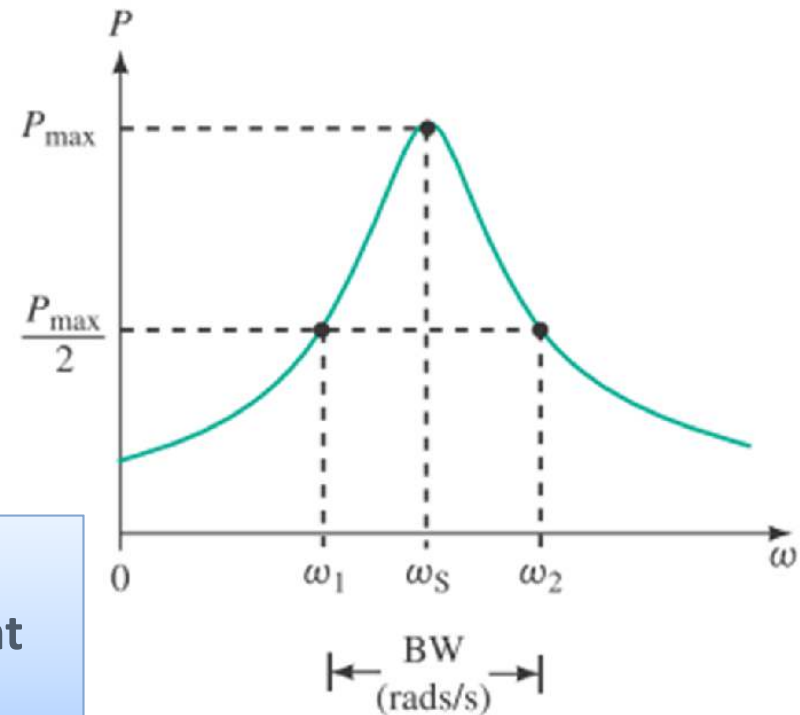
**They called the half-power frequencies (Points)**

## The Bandwidth – Selectivity – Quality Factor

Half-Power Frequencies (Points)  
Cutoff Frequencies  
Band frequencies

✓ This bell-shaped curve is called the **selectivity curve**

✓ Examining this curve, we see that only frequencies around  $\omega_s$  will permit significant amounts of power.



The Bandwidth of the resonant circuit (BW)

The difference between the frequencies at which the circuit delivers half of the maximum power.

$$BW = \omega_2 - \omega_1$$

It is called Half-Power Bandwidth

## The Bandwidth – Selectivity – Quality Factor

- ✓ If the **bandwidth** of a circuit is kept **very narrow**, the circuit is said to have a **high selectivity**,

since it is highly selective to signals within a very narrow range of frequencies.

- ✓ On the other hand, if the bandwidth of a circuit is **large**, the circuit is said to have a **low selectivity**.

### The elements of a series resonant circuit determine:

- The frequency at which the circuit is resonant
- The **shape** (and hence the **bandwidth**) of the power response curve.

1. If R and  $\omega_s$  are kept constant:

- ✓ By **increasing the ratio of L/C**, the **sides** of the power response curve become **steeper** (i.e. **decrease** in the bandwidth)
- ✓ Inversely, **decreasing** the ratio of L /C causes the sides of the curve to become more gradual (i.e. **increased** bandwidth).

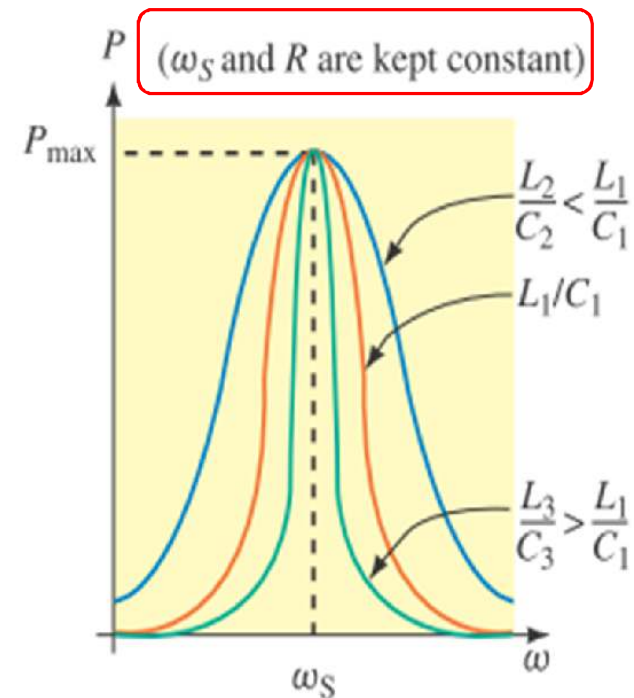
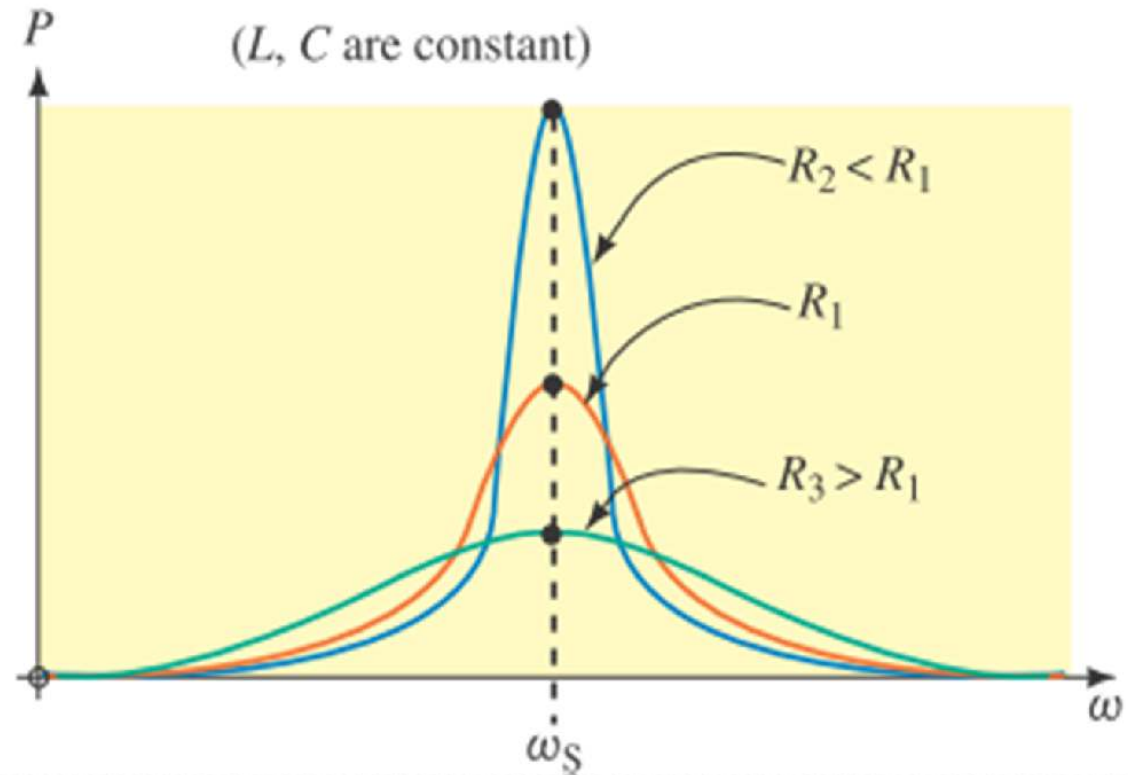


FIGURE 21-10

## The Bandwidth – Selectivity – Quality Factor

2. If L and C are kept constant:

$$I = \frac{1}{\sqrt{2}} I_m$$
$$\frac{1}{Z} = \frac{1}{\sqrt{2}} \frac{1}{R}$$
$$Z = \sqrt{2} R$$



- ✓ The **bandwidth** is directly proportional to R
- ✓ The **height** of the curve is **inversely** proportional to R

A series circuit has the **highest selectivity** if the **resistance** of the circuit is kept to a **minimum**.

## The Bandwidth – Selectivity – Quality Factor

The half-power frequencies are obtained by setting  $Z$  equal to  $\sqrt{2}R$ ,

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}R$$

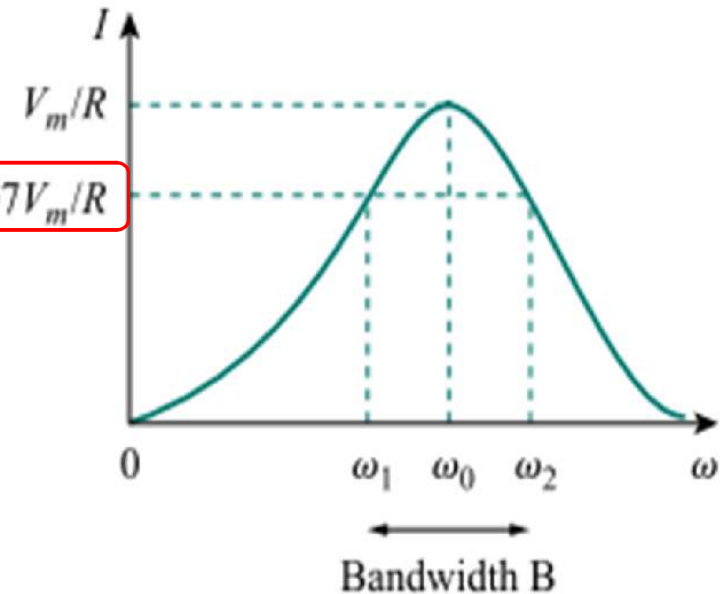
Solving for  $\omega$ , we obtain

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$
$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\text{BW} = \omega_2 - \omega_1$$
$$= \frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}} - \left(-\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}\right)$$

$$\text{BW} = \frac{R}{L} \quad (\text{rad/s})$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$



The resonant frequency is the **geometric** mean of the half-power frequencies.

## The Bandwidth – Selectivity – Quality Factor

- The “sharpness” of the resonance in a resonant circuit is measured quantitatively by the **quality factor Q**.

Q: relates the maximum or peak energy stored to the energy dissipated in the circuit per cycle of oscillation

$$Q = \frac{\text{reactive power}}{\text{average power}}$$

Notice that the quality factor is dimensionless.

$$Q_s = \frac{I^2 X_L}{I^2 R} = \frac{X_L}{R} = \frac{\omega L}{R}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$$



## The Bandwidth – Selectivity – Quality Factor

- The relationship between the bandwidth  $B$  and the quality factor  $Q$ :

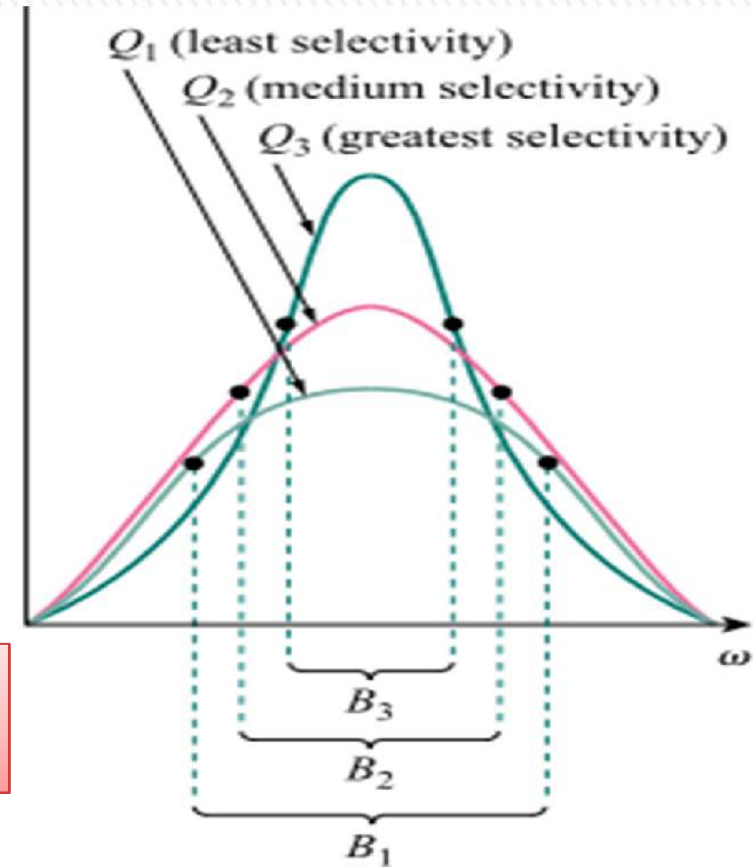
$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$$

So

$$B = \frac{R}{L} = \frac{\omega_0}{Q} = \omega_0^2 C R \quad \text{Rad/sec}$$

$$Q = \frac{\omega_0}{B}$$

The quality factor of a resonant circuit is the ratio of its resonant frequency to its bandwidth.



- The **higher** the value of  $Q$ , the **more selective** the circuit is but the **smaller** the bandwidth.

## The Bandwidth – Selectivity – Quality Factor

The selectivity of an RLC circuit is the ability of the circuit to respond to a certain frequency and discriminate against all other frequencies.

If the band of frequencies to be selected or rejected is narrow, the **quality factor of the resonant circuit must be high.**

high-Q means equal to or greater than 10.

High-Q circuits are used often in communications networks.

For high-Q, the power frequencies are, for all practical purposes, **symmetrical** around the resonant frequency and can be approximated as:

$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \quad \omega_2 \simeq \omega_0 + \frac{B}{2}$$

# ماتر (مقاومت)

[1]  $\omega_0 = \frac{1}{\sqrt{LC}}$  ,  $f_0 = \frac{1}{2\pi\sqrt{LC}}$

[2]  $Z = R + j(\omega L - \frac{1}{\omega C}) \rightarrow$  total series Res.  
 $Z = R$  (at resonance).

[3]  $BW = \omega_2 - \omega_1 = \frac{R}{L} = \frac{\omega_0}{Q}$

[4]  $\omega_{1,2} = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$   $\approx \omega_0 \pm \frac{B}{2}$  } at  $Z = \sqrt{2}R$   
 $\omega_{2,1} = \frac{+R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$   $\approx \omega_0 \pm \frac{B}{2}$  }  
 $\rightarrow$  From half power

[5]  $\omega_0 = \sqrt{\omega_1 \omega_2}$

[6]  $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$

**Thank You**

